9SG Arithmetic and Geometric Sequences and Applications Study Guide

Targets	Sample Question	Ugh	Okay	Got it	Assn
Interpret and model Arithmetic Story Problems.	 Draw a representation for 3 and 4 minutes if the pattern continues.				9A, 9B
Identify & write Arithmetic Recursive Equations	Given the sequence 12, 6, 4, 2, find the recursive equation in proper function notation.				
Identify & write Arithmetic Explicit Equations	Given the sequence 3, 6, 9, 12, find the explicit equation in proper function notation.				
Identify, and create Geometric Graphs & tables	Graph the Above questions				
Identify & write Geometric Recursive Equations	Write the recursive equation (in proper function notation) given a table				
Identify & write Geometric Explicit Equation	Write the explicit equation (in proper function notation) given a table.				
Recognize that simple interest is an example of a linear arithmetic sequence.	For a $1000 loan, Katie could not make any payments for 10 years, but she would have to pay 15% interest on the $1000 for each year of the loan. Graph				
Identify compound interest as geometric sequences/ exponential growth on a graph & table	For a $1000 loan, Katie could not make any payments for 10 years, and had to pay 10% interest on the $1000 for but the interest is compounded monthly. Create a table and graph the sequence.				
Compound Interest with an equation	Above—Write an equation to calculate much she would pay over the 10 years.				
Growth and Decay	A tarantula farm starts with 2 tarantulas that love each other very much. How many will he have after 2 years if they have 200 babies every 6 months.				
Recognize that elements of a compound interest equation.	Given the equation $f(x) = 5(1.35)^x$, find the initial investment, the growth/decay rate, and the amount of the loan after 5 years.				

While there are other kinds of sequences, this unit only covers Arithmetic and Geometric Sequences.

Vocabulary

Sequence: Set of numbers increasing of decreasing at a Comm diff or Common Ratio.

Term: Any number in a sequence.

Arithmetic Sequence: Sequence that is increasing or decreasing at a Common difference.

Geometric Sequence: Sequence that multiplied by a common ratio.

Common Difference: Amt that is added or subtracted in an arithmetic sequence

Common Ratio: Value multiplied to terms in a geometric sequence each time.

Recursive Equation: Arithmetic: $f(n)=f(n-1)+d$; Geometric: $f(n)=f(n-1)\cdot r$

Explicit Equation: Arithmetic: $f(n)=f(0)+dn$; Geometric: $f(n)=f(n-1)\cdot r^n$

Exponential: Equation or growth/decay that inc./dec by a multiplier.

Growth: 1+R, 1 plus the Rate, Geometric Common Ratio

Decay: 1−R, 1 minus the Rate, Geometric Common Ratio

Simple Interest: Dollar amt of the Simple Interest Rate of the Initial amt.

Compound Interest: Amt of interest in an account based off the Rate (%)

Arithmetic Sequence

Arithmetic sequences come from linear equations and tables. The graph of an Arithmetic Sequence is a line. An Arithmetic Sequence has a common difference (d) that increases or decreases at a Constant rate by addition or subtraction from consecutive terms. An arithmetic sequence is "proportional" if there is no vertical shift or y-intercept other than (0, 0).

Two kinds of formulas are written from a sequence: the recursive formula and the explicit formula. The recursive formula reveals how much the values change from one step to the next with a common difference.

X	f(x)
1	7
2	9
3	11
4	13
5	15

The table to the right shows a common increase (difference—d) of ______.

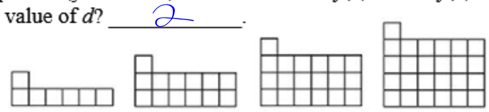
Recursive Formula

In function notation, sometime n is used instead of x. So $f(n)$ is the output when x = n. The term before n is one step before n or $(n-1)$. The output for this term is $f(n-1)$. An Arithmetic sequence changes from $f(n-1)$ to $f(n)$ by adding or subtracting a common difference (d).

The table above adds 2 for every consecutive change in x. So the recursive function of this table when x = n is $f(n) = f(n-1) + 2$. Some think of it as "What it is = what it was + the difference." To find what a step "is", 2 is added to the previous steps output.

Explicit Formula ($y = mx + b$)

An explicit formula gives the outcome for any input n. The y-intercept (where x = 0) can be written as $f(0)$. In an Arithmetic Sequence, d is the common difference, so an explicit equation ($y = mx + b$) can be written as $f(n) = dn + f(0)$.. For the table above, what is the value of $f(0)$? 5 What is the value of d? 2.



If the first figure is $f(1)$, what would be $f(0)$? ______

What would be d? ____

Write the explicit equation for the pattern. ________________ Write the recursive equation. __________________

To write an equation from a sequence, you need to know which stage the number represents. For the sequence, 5, 8, 11, 14, . . . $f(2) = 5$ means that the 2nd stage is 5. The common difference is 3, so the explicit equation would be $y = 3x - 1$ because $f(0) = -1$.

Geometric Sequence

A geometric sequence has a Common ratio "r". Multiply or divide to find the next term.

Recursive Formula

X	Y
1	4
2	8
3	16
4	32
5	64

>x 2
>x 2
>x 2
>x 2

Note that the output values double in this table. (The y-values have a common ratio of 2.)

The recursive equation for a geometric sequence can be written as $f(x) = f(x-1)(r)$. $F(x-1)$ is the prior term, and r is the common ratio. Above, the **recursive** equation in function notation would be, $f(n) = f(n-1)(2)$ or $f(n) = 2f(n-1)$.

Explicit Formula

The **common ratio** (r) is the number used to multiply an output to get the next output. This ratio is written with an exponent to show the number of times or steps (x or n) is base is multiplied. The step before x is $(x-1)$. Geometric equations often multiply the first term rather than the 0^{th} term. The explicit equation for a geometric sequence **can** be written as $f(n) = f(1)r^{(n-1)}$ where $f(n)$ is the nth term where $f(1)$ is the first term and r is the common ratio. An explicit equation could also be written using the y-intercept as $f(n) = f(0)r^{(n)}$ or from step 2 as in $f(n) = f(2)r^{(n-2)}$. The equation depends on which step the sequence begins.

X	Pattern	Y	Short Hand
1	3	3	3×2^0
2	3 x 2	6	3×2^1
3	3 x 2 x 2	12	3×2^2
4	3 x 2 x 2 x 2	24	3×2^3
5	3 x 2 x 2 x 2 x 2	48	3×2^{4n}
n	?	$f(n)$	$3 \times 2^{n-1}$

The common ratio (or multiplier r) can be seen in the 4-column table to the left. Notice the repeating multiplier is the same as the multiplier in the pattern's short hand. Note that for the nth value, **the exponent is $(n-1)$ because the table starts on step 1**.

In the short hand, note the relationship between the x (input) value and the exponent. The exponent depends on which the first input in the table. How would you write the exponent if the table started on step 2? $n-2$

Circle whether the following tables are **arithmetic or geometric.** Give the **common difference or ratio** and write the **recursive and explicit equations**.

X	1	2	3	4
Y	6	12	24	48

Arithmetic or Geometric?
Difference/Ratio: 2
Recursive: $f(x) = f(x-1) \cdot 2$
Explicit: $f(x) = 6(2)^{x-1}$

x	1	2	3	4
$f(x)$	9	27	81	243

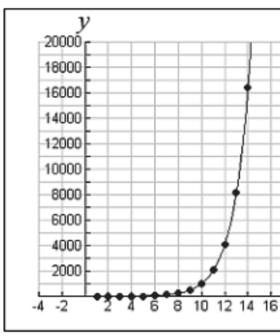
Arithmetic or Geometric?
Difference/Ratio: ________
Recursive: ______________
Explicit: ______________

x	1	2	3	4
$f(x)$	9	18	27	36

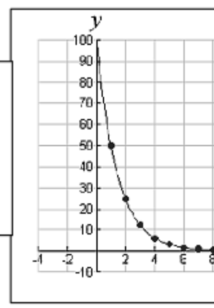
Arithmetic or Geometric?
Difference/Ratio: ________
Recursive: ______________
Explicit: ______________

Exponential Growth and Decay (Geometric Sequence)

Exponential growth and decay, occurs by a fixed percent or ratio (geometric growth or decay). For exponential growth, the rate of change increased with time – it grows faster and faster. For exponential decay, the rate of change decreased with time – the amount of decay slows down.



Growth:
When the common ratio is $r > 1$ the function is increasing (growth)
$f(x) = 1(2)^x$



Decay:
When the common ratio is $0 < r < 1$ the function is decreasing (decay)
$f(x) = 100(0.5)^x$

In order for a value to grow, a multiplier must be larger than 1. Multiplying by 1 (or 100%) would make a number or any value stay the same. The number above 1 (or 100%) indicates the percentage of the growth.

Multiply 4(1) = 4   4(1.2) = 4.8 (20% growth)   4(1.75) = 7 (75% growth)   4(2.5) = 10 (150% growth)

The explicit equation for exponential **growth** is often written $\boldsymbol{f(x) = a(1+r)^t}$ or $\boldsymbol{f(x) = f(0)(1+r)^t}$. $f(x)$ is the total amount, a or $f(0)$ is the amount of money at the start or step zero, and t is the number of compounding periods. The common ratio is r (percent of change expressed as a decimal).

A number or value will decrease if multiplied by a number less than 1. For exponential **decay**, we use the formula: $\boldsymbol{f(x) = a(1-r)^t}$.

Multiply 4(1) = 4   4(.8) = 3.2 (20% decay)   4(.25) = 1 (75% decay)   4(.05) = 0.2 (95% decay)

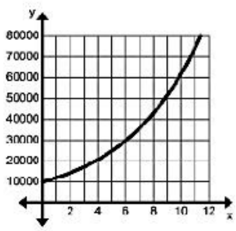
Example of explicit formulas for growth and decay would be

$f(n) = 4(1.2)^t$ would have a 20% growth $\quad$ $f(n) = 4(1.75)^t$ would have a 75% growth
$f(n) = 4(0.8)^t$ would have a 20% decay $\quad$ $f(n) = 4(0.75)^t$ would have a 25% decay.

Notice that for growth we use $(1 + R)$ and for decay we use $(1 - r)$. Why? Growth 100% of the Value plus the % of the Value. Decay is 100% of the value minus the % of the value.

Look at **GROWTH** and **DECAY** in the following situations using a graph, table, and an equation.
A business had a \$10,000 profit in 2000. Then the profit increased by 20% per year for the next 10 years.

Graph



Complete the **Table:**

Note the change in n.

$r = 1 + .2 = 1.2$

n	$f(n)$
0	\$10,000
4	20,736
6	~~\$30,000~~
9	51,597.80
10	61,917.36

\$29859.84

Equations

Explicit:

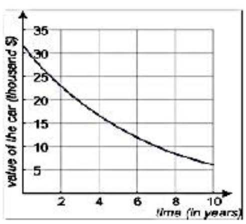
$f(t) = 10{,}000(1.20)^t$

Recursive: (write it)

$f(t) = f(t-1)(1.2)$

Look at **GROWTH** and **DECAY** in the following situations using a graph, table, and an equation.

Graph



Complete the **Table:**

Note the change in t.

t	$f(t)$
0	\$32,000
2	\$23,120
4	\$16,700
5	\$14,198.57
8	\$8719.70

Equations

Explicit:

$f(t) = 32000(0.85)^t$

Recursive: (write it)

$f(t) = f(t-1)(0.85)$

Geometric Growth (Compound Interest):

You purchase a car for \$15,000 and the loan has an interest rate of 5% compounding each year.

Make a table:

t	Pattern	$f(t)$	S.H.
0		\$15,000	
1		\$15,750	
2			

Write an **equation** for the amount of the money you owe after "t" years. ____________

If you make NO payments, what is the total amount due after eight years? __________

Geometric Decay (Compounded Loss):

Your friend purchases a car for \$15,000 and knows that his car will depreciate 5% each year in value.

Make a table:

t	Pattern	$f(t)$	S.H.
0		\$15,000	
1		\$14,250	
2			

Write an **equation** to represent the value of the car after "t" years.____________.

Estimate the value of the car after eight years.

____

Simple Interest (Arithmetic Sequence)

Not all growth is exponential. **Simple interest adds or subtracts the same value at every period**. The equation that shows that a quantity grows by the same amount at every step (or constant rate) is linear. The equation starts with the initial value (y-intercept). The amount added every period is the common difference or rate of change.

To calculate how much a value will change at each step, the initial amount will be multiplied by the percent.

For example: An investment of $3,000 is made at an annual simple interest rate of 5%. This means that $3000 is invested and it will grow 5% for every time period.

The amount of growth (or common difference/rate of change) comes from **3000 X .05 = 150** per step.

Make a table:

t	Pattern	$f(t)$	S.H.
0	3000	$3,000	3000+150(0)
1	$3,000 + $150	$3,150	3000+150(1)
2	3000+150+150	$3,300	3000+150(2)
5	3000+150+150+150 +150+150	$3750	3000+150(5)

What is the y-intercept? (0, 3000)

What is the slope? 150

Write the equation: f(t) = 3000+150t

Find how much money you would have after 8 years.
$f(8) =$ 4200

This would be an example of a(n) arithmetic sequence and the graph would be linear

Simple interest is written in the form y = mx + b where y is TOTAL AMT, m is the Interest ($) difference, x is the number of periods, and b is INITIAL AMT

**Note that Simple Interest is calculated using y = mx + b. Define the variables of this equation used to calculate simple interest.

Y = ______________________ X = ______________________

M = ______________________ B = ______________________

COMMON ERRORS:

When writing equations for simple interest, students confuse rate of change with percent growth.

In the example above, with an investment of $3,000 is made at an annual simple interest rate of 5%. This means that $3000 is invested and it will grow 5% for every time period. Students often write the equation as $f(x) = 3000 + 1.05x$.

A table reveals that instead of earning $150 per time period, the money only increases by $1.05 per period.

t	Pattern	$f(t)$	S.H.
0	3,000	$3,000	3000+1.05(0)
1	3,000 + 1.05	3001.05	3000+1.05(1)
2	3,000 + 1.05+ 1.05	3002.10	3000+1.05(2)

WRONG!!

Calculate the amount of change from the percentage **first and then add/subtract.**

Sometimes students calculate the rate of change correctly, but think they have to add a 1. This is only for geometric growth where the original amount is included in the multiplier.

February 20, 2018