## Secondary 1 Term 3

## Text \& Reference Manual

This booklet is to help bridge the gap between parents, students and teacher. We will go through some of it in-class, but it is the student's responsibility to complete.

## Teacher:

$\qquad$

## Class Period:

## Key Policies for Secondary Math 1:

> Students must pass every Unit Test with an $80 \%$ or better in order to receive a passing grade. Otherwise, grade will be an " $F$ " or an "I" as determined by the teacher.
$>$ Student may retake any Unit Test as many times as necessary to show understanding of the essential standards in the core.
$>$ Student should complete these study guides as part of the class requirement.
$>$ Homework turned in after the due date will receive a penalty to credit unless excused by the teacher.
$>$ Term finals may NOT be retaken for a higher score and must be completed in one sitting.
$>$ Each term includes a final date when homework will no longer be accepted for credit.

## Study Guide Grades

## Unit 7

Parallel Lines \& Angles Due January 11/12
Unit 8 Study Guide Function Operations
Due February 1/2
Unit 9 Study Guide
Sequences
Due February 22/23

## Please review the following policies for Secondary One:

$>$ Students must pass every Unit Test with an $80 \%$ or better in order to receive a passing grade.
> Practice tests for each unit are available online to help prepare for the tests.
$>$ Students may retake any unit test as many times as necessary to show understanding of the essential standards in the core.
$>$ Any failing grade can be made up to a passing grade until the last week of term 4.
$>$ Students may take missing tests after the end of any term as needed, but we encourage students to make up tests as soon as possible after the initial administration.
$>$ A failing grade must be made within one term to earn a grade higher than a D-. Any grade made up after one term must be by contract with the teacher or student will have an F on their permanent record and will have to make up the credit online.
$>$ Traditional textbooks are available upon request.
$>$ Term finals may not be retaken for a higher score and must be completed in one sitting unless there are extenuating circumstances are presented before the test is administered.
$>$ Homework turned in after the due date will receive a penalty to credit unless excused by the teacher because of absence or other extenuating circumstances.
$>$ Each term includes a final date when homework will no longer be accepted for credit.
$>$ If students damage a class-provided calculator (TI-84) a fee of $\$ 90$ will be added to school fees and the student will no longer will have access to a school calculator.
> If students damage a class-provided iPad, a fee of $\$ 450$ will be added to school fees and the student will no longer will have access to another calculator.
$>$ Students should complete the study guides included in this packet as part of the homework requirement: These study guides provide information on each concept tested for the unit.

TERM 3: Jan 3 - Mar 9

| UNIT 7-Parallel Lines with Angles |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Assn | Learning Objective | A Day | B Day | Done | Core Std |
| 7A | Angle Relations | Dec 15 | Dec 18 |  | G.CO.1, \& 12 |
|  |  | Dec 19 | Dec 20 | $\frac{1}{2}$ day) |  |
| 7B | All About Lines | Jan 3 | Jan 4 |  | G.CO.2, G.GPE. 5 |
| 7C | Constructing Angles | Jan 5 | Jan 8 |  | G.CO. 12 |
| 7D | Parallel Lines | Jan 9 | Jan 10 |  | G.CO. 12 |
| 7R | Unit 7 Review | Jan 11 | Jan 12 |  |  |
|  |  | OL Jan 1 |  |  |  |
|  | Unit 7 Parallel Lines Test | Jan 16 | Jan 17 |  |  |


| Unit 8-Function Operations |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Assn | Learning Objective | A Day | B Day | Core Std |
|  | Adding Functions | Jan 18 | Jan 19 | F.IF.1, 2, \& 7 |
| 8B | More Adding | Jan 22 | Jan 23 | F.IF.1, 2, 4, 5, 7, \& 9 |
| 8C | Multiplying Functions | Jan 24 | Jan 25 | F.IF.1, 2, 4, 5, 7, \& 8 |
| 8D | Shifting | Jan 26 | Jan 29 | F.IF.2, F.IF.4, F.IF.5, F.IF. 6 |
| 8R | Unit 8 Review | Jan 30 | Jan 31 |  |
|  | Unit 8 Function Operations Test | Feb 1 | Feb 2 |  |


| UNIT 9-Sequences |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Assn | Learning Objective | A Day | B Day | Core Std |
| 9A | Arithmetic Sequences | Feb 5 | Feb 6 | F.IF.3, F.BF. 1 \& 2, F.LE.2, |
| 9B | Geometric Sequences | Feb 7 | Feb 8 | F.IF.3, F.IF.6, F.BF.2, F.LE. 3 |
| 9 C | Arithmetic/Geometric and Linear/Exponential (Simple/Compound) | Feb 9 | Feb 12 | F.IF.3, F.IF.7, F.IF.9, F.BF.1, F.BF. 2 |
| 9D | Growth and Decay | Feb 13 | Feb 14 | F.LE.1-3, F.LE. 5 |
| 9E | More of everything | Feb 15 | Feb 16 | F.IF.3, F.IF.5, F.BF.1, F.BF. 2 |
| ***NO SCHOOL Feb $19{ }^{\text {th }}$ |  |  |  |  |
| 9R | Sequence Review | Feb 20 | Feb 21 |  |
|  | Sequence Test | Feb 22 | Feb 23 |  |
|  | Term 3 Final Review Work Day | Feb 26 | Feb 28 |  |
|  | Term 3 Final | Mar 1 | Mar 2 | DEAD DAY |
|  | **Professional Day ** March 6 ${ }^{\text {th }}$ |  |  |  |
|  | Remediate Day | Mar 6 | Mar 7 |  |

TERM 3 ENDS MARCH 9
ALL ASSIGNMENTS MUST BE TURNED IN BY MARCH 1 ${ }^{\text {ST }}$ (A-DAY) OR MARCH $2^{\text {ND }}$ (B-DAY) TO RECEIVE CREDIT.

## 7SG Parallel Lines Study Guide

SHOW YOUR WORK FOR FULL CREDIT. NO WORK, NO CREDIT. NO WORK IN PEN.

| Targets | Sample | Help | Not Bad | Master | Assn |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Understand congruent <br> angle relationships | Give an example of Alternate Interior Angles, <br> Same Side Exterior and Corresponding Angles <br> and state if congruent or supplementary. |  |  |  |  |
| Copy an angle | Using only a compass and straight- <br> edge, copy the following angle: |  |  |  |  |
| Construct Parallel Lines <br> with Congruent Angles | Given a line segment and point, not on the line, <br> construct a parallel line using a compass and <br> straight edge |  |  |  |  |
| Using angle relationships <br> to find the measure of <br> angles. | If angles $a$ and $b$ are corresponding and the <br> measure of angle $a=4+2 p$ and $b=8 p-14$, <br> find $p$ and the measure of $a$ and $b$. |  |  |  |  |

## Vocabulary

Parallel Lines:
Perpendicular Lines: $\qquad$
Right Angle: $\qquad$
Transversal: $\qquad$
Interior: $\qquad$
Exterior: $\qquad$
Adjacent: $\qquad$
Supplementary Angles: $\qquad$
Complementary Angles:
IF $\boldsymbol{l} \boldsymbol{\|} \boldsymbol{m}$ in the following image, give an example of each kind of angle:
Vertical Angles: $\angle A$ \& $\qquad$ Corresponding Angles: $\angle D$ \&
Same-Side Interior Angles: $\angle C$ \& $\qquad$ Same-Side Exterior Angles: $\angle H$ \& $\qquad$ Alternate Interior Angles: $\angle D$ \& $\qquad$ Alternate Exterior Angles: $\angle H$ \& $\qquad$ $\angle A$ and $\angle G$ are $\qquad$ $\angle H$ and $\angle D$ are $\qquad$ $\angle D$ and $\angle E$ are $\qquad$


## Finding angle measurements

1. If lines $l$ and $m$ are parallel, $\angle A$ and $\angle E$ are $\qquad$ angles and their measurements are
$\qquad$ . If $\angle A=3 \mathrm{x}+20$ and $\angle E=2 \mathrm{x}+60$, find x . $\qquad$

What is the measure of $\angle A$ ? $\qquad$ $\angle E$ ? $\qquad$
2. If lines $l$ and $m$ are parallel, $\angle H$ and $\angle B$ are $\qquad$ exterior angles and their measurements are
$\qquad$ . If $\angle H=\mathrm{x}+40$ and $\angle B=-2 \mathrm{x}+25$, find x . $\qquad$

What is the measure of $\angle H$ ? $\qquad$ $\angle B$ ? $\qquad$

Copy an Angle: Instructions on how to copy angle BAC
You can see a live animation at: http://www.mathopenref.com/constcopyangle.html

| You can see a live animation at: http://www.mathopenref.com/constcopyangle.html |  | $\qquad$ |
| :---: | :---: | :---: |
| Step 1: Make a point $P$ to be the vertex of the new $\qquad$ <br> Step 2: From $P$, draw a ray $P Q$. This will become one $\qquad$ of the new angle. | P |  |
| Step 3: Place the compass on point A and set it to any $\qquad$ <br> Step 4: Draw an $\qquad$ across both sides of the angle - mark the points $J$ and $K$ as shown. $\overline{A J}$ and $\overline{A K}$ are $\qquad$ of the same circle. |  |  |
| Step 5: Without changing the width of the $\qquad$ , place its point on $P$ and draw a congruent $\qquad$ , creating point $M$ as shown right. |  |  |
| Step 6: Measure the $\qquad$ from $K$ to $J$. <br> Step 7: Without changing the compass width, measure the same distance from point $M$ across the $\qquad$ . (The third side of congruent triangles.) |  |  |
| Step 8: Draw a ray from P through L -exact length in not important since you are only copying <br> one $\qquad$ <br> Done: $\angle J A K \cong($ congruent $) \angle L P M$ |  |  |

Follow the step above to practice copying the angles below onto the given rays. Show all markings.


Constructing a Parallel Line Through a Point. (animation at http://www.mathopenref.com/constparallel.http) (Parallel to line PQ, through point $R$ )
Step 1: Draw a segment through point R that $\qquad$ the line $P Q$ at any angle. Mark point $J$ where it intersects the line $P Q$.
Step 2: Set the width of the $\qquad$ to any length between point $R$ and $J$.
Draw an $\qquad$ across lines $\overline{R J}$ and $\overline{P Q}$ at $J$.
Step 3: Without changing the compass $\qquad$ draw a congruent $\qquad$ at point
$R$ in the same orientation as the arc in Step 2.
Step 4: Measure the distance from X to S .


Step 5: Copy that same distance from $r$ to the lower arc intersection.
Step 6: Because the corresponding angles $\angle R J Q$ and $\angle X R S$ are congruent, lines $\overleftrightarrow{R S}$ and $\overleftrightarrow{P Q}$ are parallel. Construct a line parallel to the line below that passes through the given point. Show All Markings.


## 8SG Function Operations Study Guide

| Targets | Sample Question | Struggle | Meh | Yeah! | Assn |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Add and Subtract Functions | Given $\mathrm{f}(\mathrm{x}) \& \mathrm{~g}(\mathrm{x})$, find $\mathrm{f}(\mathrm{x})+\mathrm{g}(\mathrm{x})$ <br> algebraically and graphically by hand and by <br> technology. Show how with a table. |  |  | 8 A, <br> 8 B |  |
| Multiply Expressions | Give $f(\mathrm{x})=3 \mathrm{x}+5$ and $g(\mathrm{x})=5 \mathrm{x}+5$. <br> Find $f(\mathrm{x}) g(\mathrm{x})$ |  |  | 8 B, <br> 8 C, |  |
| Shifts (Vertical) | Given an equation, explain what would happen <br> if $\mathrm{f}(\mathrm{x})$ changes to $\mathrm{f}(\mathrm{x})+4$. |  |  | 8 D, <br> 8 R |  |

## Vocabulary

## Parabola:

## Binomial

Vertical Shift:
Horizontal Shift:
Vertical Stretch:

## Adding/Subtracting Functions

Lines have only one dimension (width or height) but not $\qquad$ . Adding or subtracting lines only changes how it looks, but does not change that they are lines. Adding lines creates a new $\qquad$ . The input ( x ) gives an output $f(\mathrm{x})$. Linear outputs can be added which give the same equation as adding the two linear functions.

| x | $f(\mathrm{x})$ | $g(\mathrm{x})$ | $f(\mathrm{x})+g(\mathrm{x})$ | $f(\mathrm{x})-g(\mathrm{x})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 6 |  | 12 | 0 |
| 2 | 9 | 8 |  | 1 |
| 3 | 12 |  | 22 | 2 |
| 4 |  | 12 |  | 3 |
| 5 |  | 14 | 32 |  |

Fill in the missing values on the table to the right then use the table above to fill in the table below.

|  | Slope | Y-int | Equation |
| :---: | :--- | :--- | :--- |
| $f(\mathrm{x})$ |  |  |  |
| $g(\mathrm{x})$ |  |  |  |
| $f(\mathrm{x})+g(\mathrm{x})$ |  |  |  |
| $f(\mathrm{x})-g(\mathrm{x})$ |  |  |  |

Graph the equations on the grid. Note that the points in the table can be added or subtracted just like the $y$-values on a graph to graph the sum or difference of the functions.

## Transformations

The "parent graph of a linear equation is $\mathrm{y}=\mathrm{x}$. (In the parent equation, the slope is $\qquad$ and the y-intercept is $\qquad$


To shift the parent equation vertically (up or down), add or $\qquad$ a y-intercept. From the parent graph, write the equation with a vertical shift of +9 . $\qquad$
Slope is used to show the steepness of the line/graph. When the slope is constant, the graph will be $\qquad$ . If the graph is not a line, there can still be a "stretch" over an interval. Another name for slope, then, is "vertical $\qquad$ $"$ as the rise is "stretched" or "smooshed" compared to the slope of $1 / 1$ in the parent equation. For the equation $y=3 x-6$, the vertical stretch would be $\qquad$ .

The vertical shift and vertical stretch are very obvious in $\mathrm{y}=\mathrm{mx}+$ $\qquad$ form. Like the vertical shift, the horizontal (left or right) shift can be seen in the equation if the slope is factored out of the equation. $\left(\mathrm{y}=\mathrm{m}\left(\mathrm{x}+\frac{b}{m}\right)\right.$.


For example: in the equation $y=3 x-6$ graphed above, the vertical shift is -6 and the vertical stretch is $\qquad$ . Factor out the stretch and the equation becomes $y=3(x-2)$. The $x$-intercept is $(2,0)$ and the horizontal shift is 2 .

HINT: after factoring out the slope/stretch, the shift has the opposite sign of the number in the $\qquad$ . In a linear equation, this is also the x -

## Multiplying Functions

Multiplying two one-dimensional figures (linear equations) results in a two dimensional figure. ("When you multiply, you
$\qquad$ the dimensions.") This results in a dramatic change in your data. Complete the table.

In the table, notice that the outputs repeat when the equations are multiplied.
Find the equation for $f(\mathrm{x})$ : $\qquad$ Find the equation for $g(\mathrm{x})$ : $\qquad$
What is the vertical shift of $f(x)$ ? $\qquad$ What is the vertical stretch of $f(\mathrm{x})$ ? $\qquad$ Write the equation for $f(\mathrm{x})$ that exposes the horizontal shift: $\qquad$

| x | $f(\mathrm{x})$ | $g(\mathrm{x})$ | $f(\mathrm{x}) g(\mathrm{x})$ |
| :---: | :---: | :---: | :---: |
| -5 | -9 | -1 | 9 |
| -4 |  | 0 | 0 |
| -3 | -3 |  | -3 |
| -2 | 0 | 2 |  |
| -1 |  | 3 | 9 |

What is the vertical shift of $g(x)$ ? $\qquad$ What is the vertical stretch of $g(\mathrm{x})$ ? $\qquad$
Write the equation for $g(\mathrm{x})$ that exposes the horizontal shift: $\qquad$
Set up the equation for $f(\mathrm{x}) g(\mathrm{x})$ that shows the two equations (factors) being multiplied. $\qquad$
Stacking Method: This method uses basic multiplication practices. To multiply 24 X 18,

1. Stack one binomial on top of the other. $x+4$

## $\underline{x+8}$

2. Multiply the same as multiplying multi-digit numbers like 24 X 18.

## Box Method

$$
\begin{array}{r}
x+4 \\
x+8 \\
\hline 8 x+32 \\
\frac{x^{2}+4 x}{x^{2}+12 x+32}
\end{array}
$$



1. Think of each term like the sides of a $\qquad$ . For $(x+4)(x+8),(x+4)$ could be its length and $(x+8)$ could be the width.
2. Split the binomials to represent each $\qquad$ .
3. Multiply to $\qquad$ the area of the smaller boxes.

Add like terms in your box together: $\mathrm{x}^{2}+12 \mathrm{x}+32$.

## Distributive Method



1. A number next to a parenthesis means to multiply. Multiply every term each term in the first set of parentheses by each term in the second set of parentheses.
2. 


$=x(x-5)+4(x-5)$ Distribute over addition $=\mathrm{x}^{2}-5 \mathrm{x}+4 \mathrm{x}-20$ and combine like terms, so $=\mathrm{x}^{2}-\mathrm{x}-20$
Use TWO methods from above to multiply and check your answers (SYW)

1. $(x+6)(x+9)$
2. $(x+3)(x-1)$
3. $(x+5)(2 x+2)$

## 9SG Arithmetic and Geometric Sequences and Applications Study Guide

| Targets | Sample Question | Ugh | Okay | Got it | Assn |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Interpret and model Arithmetic Story Problems. | $\bullet$ $\because$ $\ddots \circ^{\circ}$ Draw a representation for 3 and <br> 4 minutes if the pattern <br> continues.    |  |  |  | $\begin{aligned} & \text { 9A, } \\ & \text { 9B } \end{aligned}$ |
| Identify \& write Arithmetic Recursive Equations | Given the sequence $12,6,4,2$, find the recursive equation in proper function notation. |  |  |  |  |
| Identify \& write Arithmetic Explicit Equations | Given the sequence $3,6,9,12$, find the explicit equation in proper function notation. |  |  |  |  |
| Identify, and create Geometric Graphs \& tables | Graph the Above questions |  |  |  |  |
| Identify \& write Geometric Recursive Equations | Write the recursive equation (in proper function notation) given a table |  |  |  |  |
| Identify \& write Geometric Explicit Equation | Write the explicit equation (in proper function notation) given a table. |  |  |  |  |
| Recognize that simple interest is an example of a linear arithmetic sequence | For a $\$ 1000$ loan, Katie could not make any payments for 10 years, but she would have to pay $15 \%$ interest on the $\$ 1000$ for each year of the loan. Graph |  |  |  |  |
| Identify compound interest as geometric sequences/ exponential growth on a graph \& table | For a $\$ 1000$ loan, Katie could not make any payments for 10 years, and had to pay $10 \%$ interest on the $\$ 1000$ for but the interest is compounded monthly. Create a table and graph the sequence. |  |  |  |  |
| Compound Interest with an equation | Above-Write an equation to calculate much she would pay over the 10 years. |  |  |  |  |
| Growth and Decay | A tarantula farm starts with 2 tarantulas that love each other very much. How many will he have after 2 years if they have 200 babies every 6 months. |  |  |  |  |
| Recognize that elements of a compound interest equation. | Given the equation $f(x)=5(1.35)^{x}$, find the initial investment, the growth/decay rate, and the amount of the loan after 5 years. |  |  |  |  |

While there are other kinds of sequences, this unit only covers Arithmetic and Geometric Sequences.

## Vocabulary

Sequence:
Term:
Arithmetic Sequence:
Geometric Sequence: $\qquad$
Common Difference: $\qquad$
Common Ratio:
Recursive Equation: $\qquad$
Explicit Equation: $\qquad$
Exponential: $\qquad$
Growth: $\qquad$
Decay: $\qquad$
Simple Interest: $\qquad$

## Compound Interest:

## Arithmetic Sequence

Arithmetic sequences come from $\qquad$ equations and tables. The graph of an Arithmetic Sequence is a $\qquad$ . An Arithmetic Sequence has a common difference ( $d$ ) that increases or decreases at a
$\qquad$ rate by addition or subtraction from consecutive terms. An arithmetic sequence is "proportional" if there is no vertical shift or $y$-intercept other than $(0,0)$.

Two kinds of formulas are written from a sequence: the $\qquad$ formula and the explicit formula. The recursive formula reveals how much the values change from one step to the next with a $\qquad$ difference.

The table to the right shows a common increase (difference-d) of $\qquad$ .

| X | $\mathrm{f}(\mathrm{x})$ |
| :---: | :---: |
| 1 | 7 |
| 2 | 9 |
| 3 | 11 |
| 4 | 13 |
| 5 | 15 |

## Recursive Formula

In function notation, sometime $n$ is used instead of x . $\operatorname{Sof} f(n)$ is the output when $\mathrm{x}=\mathrm{n}$. The term before $n$ is one step before $n$ or $(n-1)$. The output for this term is $f(n-1)$. An Arithmetic sequence changes from $f(\mathrm{n}-1)$ to $f(\mathrm{n})$ by adding or subtracting a common difference ( $d$ ).

The table above adds 2 for every consecutive change in x . So the recursive function of this table when $\mathrm{x}=\mathrm{n}$ is $f(n)=f(n-1)+2$. Some think of it as "What it is = what it was + the difference." To find what a step "is", 2 is added to the previous steps output.

## Explicit Formula ( $\mathrm{y}=\mathrm{mx}+\mathrm{b}$ )

An explicit formula gives the outcome for any input $n$. The y-intercept (where $\mathrm{x}=0$ ) can be written as $f(0)$. In an Arithmetic Sequence, $d$ is the common difference, so an explicit equation ( $\mathrm{y}=\mathrm{mx}+\mathrm{b}$ ) can be written as $f(\mathrm{n})=d n+f(0)$.. For the table above, what is the value of $f(0)$ ? $\qquad$ What is the value of $d$ ? $\qquad$ .

If the first figure is $f(1)$, what would be $f(0)$ ? $\qquad$
What would be $d$ ? $\qquad$


Write the explicit equation for the pattern. $\qquad$ Write the recursive equation. $\qquad$

To write an equation from a sequence, you need to know which stage the number represents. For the sequence, 5, 8, 11, $14, \ldots f(2)=5$ means that the $\qquad$ stage is 5 . The common difference is $\qquad$ , so the explicit equation would be $\mathrm{y}=3 \mathrm{x}-1$ because $. f(0)=$ $\qquad$ .

## Geometric Sequence

A geometric sequence has a $\qquad$ ratio " $r$ ". Multiply or divide to find the next $\qquad$ .

## Recursive Formula

| X | Y |
| :---: | :---: |
| 1 | 4 |
| 2 | 8 |
| 3 | 16 |
| 4 | 32 |
| 5 | 64 |

Note that the output values double in this table. (The y -values have a common ratio of 2.)

The recursive equation for a geometric sequence can be written as $f(x)=f(x-1)(r) . F(x-1)$ is the prior term, and r is the common ratio. Above, the recursive $\qquad$ in function notation would be, $f(n)=f(n-l)(2)$ or $f(n)=2 f(n-1)$.

## Explicit Formula

The common ratio $(r)$ is the number used to multiply an output to get the next output. This ratio is written with an
$\qquad$ to show the number of times or steps $(x$ or $n)$ is base is multiplied. The step before $x$ is $(x-1)$. Geometric equations often multiply the first term rather than the $0^{\text {th }}$ term. The explicit $\qquad$ for a geometric sequence can be written as $f(\mathrm{n})=f(1) \mathrm{r}^{(\mathrm{n}-1)}$ where $f(\mathrm{n})$ is the nth term where $f(1)$ is the first term and r is the
common ratio. An explicit equation could also be written using the y-intercept as $f(\mathrm{n})=f(0) \mathrm{r}^{(\mathrm{n})}$ or from step 2 as in $f(\mathrm{n})=f(2) \mathrm{r}^{(\mathrm{n}-2)}$. The equation depends on which step the sequence begins.

| X | Pattern | Y | Short Hand |
| :---: | :---: | :---: | :---: |
| 1 | 3 | 3 | $3 \times 2^{0}$ |
| 2 | $3 \times 2$ | 6 | $3 \times 2^{1}$ |
| 3 | $3 \times 2 \times 2$ | 12 | $3 \times 2^{2}$ |
| 4 | $3 \times 2 \times 2 \times 2$ | 24 | $3 \times 2^{3}$ |
| 5 | $3 \times 2 \times 2 \times 2 \times 2$ | 48 | $3 \times 2^{4 \mathrm{n}}$ |
| $n$ | $?$ | $f(n)$ | $3 \times 2^{\mathrm{n}-1}$ |

The common ratio (or multiplier $r$ ) can be seen in the 4column table to the left. Notice the repeating multiplier is the same as the multiplier in the pattern's short $\qquad$ . Note that for the nth value, the exponent is $(\boldsymbol{n}-\mathbf{1})$ because the table starts on step 1.

In the short $\qquad$ , note the relationship between the x (input) value and the exponent. The exponent depends on which the first input in the table. How would you write the exponent if the table started on step 2 ?

Circle whether the following tables are arithmetic or geometric. Give the common difference or ratio and write the recursive and explicit equations.

| X | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| Y | 6 | 12 | 24 | 48 |

Arithmetic or Geometric?
Difference/Ratio: $\qquad$
Recursive: $\qquad$
Explicit:

| $x$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 9 | 27 | 81 | 243 |

Arithmetic or Geometric?
Difference/Ratio: $\qquad$
Recursive:
Explicit:

| $x$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 9 | 18 | 27 | 36 |

Arithmetic or Geometric?
Difference/Ratio: $\qquad$
Recursive: $\qquad$
Explicit: $\qquad$

## Exponential Growth and Decay (Geometric Sequence)

Exponential growth and decay, occurs by a fixed percent or ratio (geometric growth or decay). For exponential growth, the rate of change $\qquad$ with time - it grows faster and faster. For exponential $\qquad$ , the rate of change decreased with time - the amount of decay slows down.


In order for a value to grow, a multiplier must be larger than $\qquad$ . Multiplying by 1 (or $100 \%$ ) would make a number or any value stay the same. The number above 1 (or $100 \%$ ) indicates the percentage of the growth.

Multiply 4(1) = ___

$$
4(1.2)=
$$

$\qquad$ $(20 \%$ growth $) \quad 4(1.75)=$ $\qquad$ ( $75 \%$ growth $) \quad 4(2.5)=$ $\qquad$ (150\% growth)

The explicit equation for exponential growth is often written $\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{a}(\mathbf{1}+\boldsymbol{r})^{\mathrm{t}}$ or $\boldsymbol{f}(\boldsymbol{x})=\boldsymbol{f}(\mathbf{0})(\mathbf{1}+\boldsymbol{r})^{\boldsymbol{t}} . f(x)$ is the total amount, $a$ or $f(0)$ is the amount of money at the start or step zero, and $t$ is the number of compounding periods. The common ratio is $r$ (percent of change expressed as a decimal).

A number or value will decrease if multiplied by a number less than $\qquad$ . For exponential decay, we use the formula: $f(x)=a(1-r)^{\mathrm{t}}$.

Multiply 4(1) = $\qquad$

$$
4(.8)=\ldots \quad(20 \% \text { decay })
$$

$4(.25)=$ $\qquad$ (75\% decay)

$$
4(.05)=
$$

$\qquad$ (95\% decay)

Example of explicit formulas for growth and decay would be

$$
\begin{array}{ll}
f(n)=4(1.2)^{t} \text { would have a } 20 \% \text { growth } & f(n)=4(1.75)^{t} \text { would have a } 75 \% \text { growth } \\
f(n)=4(0.8)^{t} \text { would have a } 20 \% \text { decay } & f(n)=4(0.75)^{t} \text { would have a } 25 \% \text { decay. }
\end{array}
$$

Notice that for growth we use $(1+r)$ and for $\qquad$ we use $(1-r)$. Why? $\qquad$

Look at GROWTH and DECAY in the following situations using a graph, table, and an equation.
A business had a $\$ 10,000$ profit in 2000 . Then the profit increased by $20 \%$ per year for the next 10 years.

Complete the Table:

| Note the <br> change in $n$. |
| :--- | | $n$ | $f(n)$ |
| :---: | :---: |
| 0 | $\$ 10,000$ |
| 4 |  |
| 6 | $\$ 30,000$ |
| 9 |  |
| 10 |  |

## Equations

## Explicit:

$f(t)=10,000(1.20)^{t}$
Recursive: (write it)
$f(t)=$
Look at GROWTH and DECAY in the following situations using a graph, table, and an equation.

| Graph | Complete the Table: |  |  | Equations |
| :---: | :---: | :---: | :---: | :---: |
| $\hat{5}^{35} \mid$ | Note the | $t$ | $f(t)$ | Explicit: |
| 频25 | change in $t$. | 0 | \$32,000 |  |
| 흔 20 |  | 2 |  | $f(t)=$ |
| ${ }_{5}^{\circ}$ |  | 4 | \$16,700 |  |
| ${ }_{\text {\% }}{ }_{5}^{10}$ |  | 5 |  | Recursive: (write it) |
|  |  | 8 |  | $f(t)=f(t-1)(0.85)$ |

## Geometric Growth (Compound Interest):

You purchase a car for $\$ 15,000$ and the loan has an interest rate of $5 \%$ compounding each year.
Make a table:

| $t$ | Pattern | $f(t)$ | S.H. |
| :---: | :--- | :---: | :--- |
| 0 |  | $\$ 15,000$ |  |
| 1 |  | $\$ 15,750$ |  |
| 2 |  |  |  |

Write an equation for the amount of the money you owe after " $t$ " years.
If you make NO payments, what is the total amount due after eight years? $\qquad$

## Geometric Decay (Compounded Loss):

Your friend purchases a car for $\$ 15,000$ and knows that his car will depreciate $5 \%$ each year in value.
Make a table:

| $t$ | Pattern | $f(t)$ | S.H. |
| :---: | :--- | :---: | :--- |
| 0 |  | $\$ 15,000$ |  |
| 1 |  | $\$ 14,250$ |  |
| 2 |  |  |  |

Write an equation to represent the value of the car after " $t$ " years.
Estimate the value of the car after eight years.

## Simple Interest (Arithmetic Sequence)

Not all growth is exponential. Simple interest adds or subtracts the same value at every period. The equation that shows that a quantity grows by the same amount at every step (or constant rate) is $\qquad$ . The equation starts with the initial value (y-intercept). The amount added every period is the common difference or rate of change.

To calculate how much a value will change at each step, the initial amount will be multiplied by the percent.
For example: An investment of $\$ 3,000$ is made at an annual simple interest rate of $5 \%$. This means that $\$ 3000$ is invested and it will grow $5 \%$ for every time period.

The amount of growth (or common difference/rate of change) comes from $\mathbf{3 0 0 0} \mathbf{X} \mathbf{. 0 5}=\mathbf{1 5 0}$ per step.

## Make a table:

| $t$ | Pattern | $f(t)$ | S.H. |
| :--- | :--- | :---: | :--- |
| 0 |  | $\$ 3,000$ |  |
| 1 | $\$ 3,000+\$ 150$ |  |  |
| 2 |  | $\$ 3,300$ |  |
| 5 |  |  |  |

What is the y-intercept? $\qquad$
What is the slope? $\qquad$
Write the equation: $\qquad$
Find how much money you would have after 8 years. $f(8)=$ $\qquad$
This would be an example of $a(n)$ $\qquad$ sequence and the graph would be $\qquad$
Simple interest is written in the form $\mathbf{y}=\mathbf{m x}+\mathbf{b}$ where y is $\qquad$ , $m$ is the $\qquad$ difference, $x$ is the number of $\qquad$ and $b$ is $\qquad$
**Note that Simple Interest is calculated using $\mathrm{y}=\mathrm{mx}+$ $\qquad$ . Define the variables of this equation used to calculate simple interest.
$\qquad$
$\mathrm{X}=$ $\qquad$
B = $\qquad$
$\mathrm{M}=$ $\qquad$

## COMMON ERRORS:

When writing equations for simple interest, students confuse rate of change with percent growth.

In the example above, with an investment of $\$ 3,000$ is made at an annual simple interest rate of $5 \%$. This means that $\$ 3000$ is invested and it will grow $5 \%$ for every time period. Students often write the equation as $f(x)=3000+1.05 x$.

A table reveals that instead of earning $\$ 150$ per time period, the money only increases by $\$$ $\qquad$ per period.

| $t$ | Pattern | $f(t)$ | S.H. |
| :---: | :--- | :---: | :--- |
| 0 | 3,000 | $\$ 3,000$ |  |
| 1 | $3,000+1.05$ |  |  |
| 2 | $3,000+1.05+1.05$ |  |  |

## Calculate the amount of change from the percentage first and then add/subtract.

Sometimes students calculate the rate of change correctly, but think they have to add a 1 . This is only for geometric growth where the original amount is included in the multiplier.

